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FINAL REPORT

Jet Propulsion Laboratory
~~Contract No.~~ 951100

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1. STATEMENT OF THE PROBLEM

The purpose of the study which this report describes was to determine if the incorporation of certain features in the design of free space rooms would improve their electrical performance and if so to determine their proper design. These features which we give the generic name stereometry, include longitudinal baffles, transverse irises, pyramidal back walls and tilted back walls. In every case we assume a given box-shaped room whose six walls (four side walls, one front wall and one back wall) will be completely covered with a high quality absorber. If any of these features are incorporated they must protrude into the given room volume. We assume a fixed transmitting antenna of good design on or close to the front wall and a receiving or test antenna which may be rotated in any position at a point near the back wall.

The study is specifically aimed at the design of two free space rooms for JPL one to be used at 8gc of size 20' x 20' x 60' and the second to be used at 16gc of size 20' x 20' x 40'. These rooms are to be used for the measurements of high performance and generally low gain antennas.

The present study included both theoretical and experimental investigations (the latter at a scale frequency of 96gc) and is similar in approach to a previous study described in ref[1] and [2]. In the previous study the reflections due to longitudinal metal baffles and transverse metal fences were determined from asymptotic evaluation of the physical optics formulas for scattering from these obstacles on a ground plane.

If the metallic walls are replaced by absorbing walls, which are homogeneous and isotropic or equivalently may be characterized by a reflection coefficient then the formulas given in [1] and [2] may be readily modified to apply to absorbing walls as well as perfectly conducting walls.

Let us consider a free space room, initially a simple rectangular parallelepiped, of dimension A x B x C, with walls completely covered with homogeneous, isotropic absorber containing a source and a receiver in the usual locations: approximately centered with respect to the transverse A and B dimensions and a distance of from .5C to .9C apart with respect to the C or longitudinal dimension. In this case to first approximation the energy interfering with the directly transmitted energy may be associated with rays from the source which reach the receiver on one bounce off of each wall. The energy associated with them is reduced in intensity relative to the direct ray by a factor

$$(1) \qquad \frac{G_{Tr} G_{Rr} RD}{G_{Td} G_{Rd}}$$

where G_{Tr}/G_{Rd} is the ratio of the transmitter gain in the reflected ray direction to the direct ray direction, G_{Rr}/G_{Rd} is the same for the receiver, R is the wall power reflection coefficient at the incidence angle and polarization of the incident ray and D is the ratio of reflected to direct path distances squared. This energy is a source of error and is to be reduced as much as possible. While the transmitting (or fixed) antenna may be designed so that G_{Tr}/G_{Td} is small, say -20db or less except on the back wall where it will be unity, the receiving (or test) antenna may be rotated so that G_{Rr}/G_{Rd} is very large, essentially without limit. R may range from -30 to -70db depending on absorber quality and incidence angle and polarization. D is less than unity and is of the order of 2 to 6db.

The purpose of any feature such as baffles, irises, pyramids or rotation of a wall is to eliminate this specular reflection energy, replacing it in most cases with diffraction energy hopefully of less intensity. A useful way of viewing this process is that the absorbing wall is moved or shaped in such a way as to distort the reflected phase front by spoiling the phase of the first (or first few) Fresnel zones surrounding each stationary phase point, destroying or reducing its coherence.

One manner in which this has been done in all high quality free space rooms is by the use of dentated or physically tapered absorber as shown in Figure 1. This method is superior to baffles and irises in one way: it does not protrude much into the room; protrusion not only increases D in (1) but also R which typically increases with the increasing incidence angle accompanying protrusion. It is also superior in two other ways. As shown in Figure 1 much of the energy reflected from this type of absorber strikes the same wall again and perhaps several times (ray no. 1) and secondly, some of it (ray no. 2) is better absorbed because of the physical tapering being somewhat equivalent to electrical tapering. The latter type absorption is more significant at low frequencies, the former at high frequencies.

The physically tapered absorber, however, does not in principal completely break up the Fresnel zones around a stationary phase point, since due to its planar periodicity, whatever reflection occurs per period of dentation

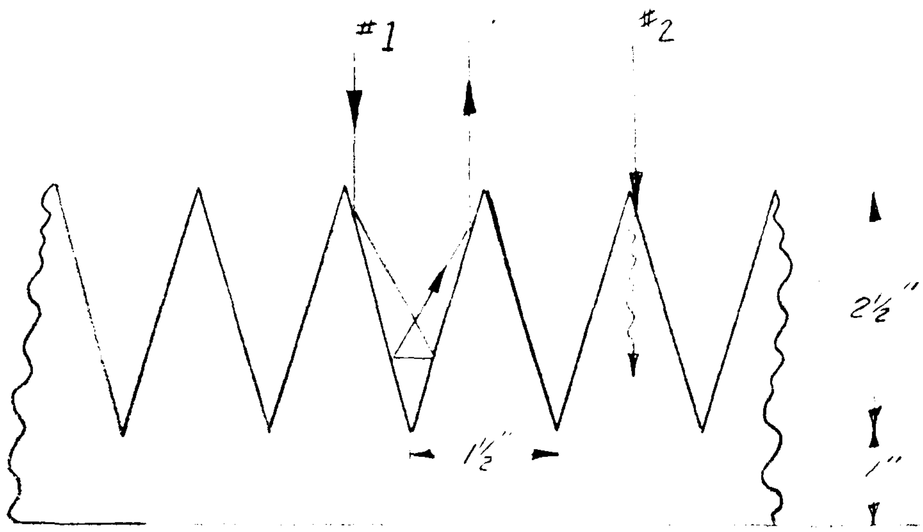

FIGURE 1

Rays Incident on Dentated Absorber

will be cophasal with the reflections from all other periods in each Fresnel zone. If this residual reflection is large enough it may be reduced further by changing the shape of the surface.

For any given observation and source points there is an ellipsoidal shaped first Fresnel zone surrounding the stationary phase point on each wall. Suppose the observation and source points are back a distance H from a side-wall and are separated by a distance r. Then the dimensions of the first Fresnel zone if $r >> H >> \lambda$ are given in Table 1.

Note that in the first Fresnel zone in the 8gc case there are of the order of 900 and in the 16gc case 180 teeth (or pyramids) of $1\frac{1}{2}$" x $1\frac{1}{2}$" period absorber. In addition the height of each tooth is several times that of the first Fresnel zone.

In the case of the front and back wall, if source and observation points are distances d and r + d from these, the first Fresnel zone has dimensions given in Table 2.

In this case the Fresnel zone contains about one period if d = 0" (antenna

First Fresnel Zones	H = 120" r = 54' λ = 1.48"	r = 34' λ = .72"
length: $\sqrt{\lambda r}\ (r/2H)$	83.6"	29.6"
width: $\sqrt{\lambda r}$	31"	17.4"
height: $\lambda r/8H$	1.0"	.3"

TABLE 1

Side Wall Fresnel Zones

	r = 54' λ = 1.48" d = 6'	d = 0	r = 34' λ = .72" d = 6'	d = 0
diameter: $\begin{cases} \lambda, & d \simeq 0 \\ 2\sqrt{\dfrac{\lambda d(r+d)}{r+2d}} & d > \lambda \end{cases}$	19.7"	1.44"	6.8"	.72"
height: $\lambda/4$	.36"	.36"	.36"	.19"

TABLE 2

Front and Back Wall Fresnel Zones

flush with front wall) there is no need to attempt to break up the early Fresnel zones with additional variations. In the case of d = 6', there are of the order of 130 teeth in the 8gc case and 9 teeth in the 16gc case. In addition the height of each tooth is many times the height of the first Fresnel zone in both cases.

Hence in all cases, the dentated absorber is a very rough surface in the sense of p. 241 of [3], and in all cases save one (d = 0) it is worthwhile to consider whether some further surface stereometric treatment can reduce the wall reflection even below that of the planar wall covered by the dentated absorber.

It is worthwhile to consider the possible improvement achievable by this means. If the test antenna has moderate or low gain in the worst case, when its peak illumination is centered on a stationary phase point, the beam dimensions on the wall are roughly of the order of the first Fresnel zone or larger. As the gain of the test antenna increases the illuminated region of the wall decreases until the limit is reached where the antenna no longer satisfies the $2D^2/\lambda$ far field criterion within the room. These limits are shown in Table 3. In no case is the illuminated region of the wall less than the area of the aperture achieving the maximum gain, and covering a large number of tooth periods of the absorber.

If the absorber were totally a rough surface scatterer, in addition to reducing the reflected energy by the mechanisms of Figure 1 and hence reducing R in (1) it may also reduce the effective gain of the test antenna G_{Rr} in the specular reflection direction. The limit is approximately to reduce its value to the isotropic level. This is generally the best that can be done by stereometry. With a low gain feed this may not be appreciable. But from Table 3 it may reach a significant value in the limiting case of 34 or 35db.

f = 8gc,	60 ft room:	$D \leq 23''$,	$G \leq 34$db
f = 16gc,	40 ft room:	$D \leq 13''$,	$G \leq 35$db

TABLE 3

Maximum Antenna Sizes and Gains ($2D^2/\lambda$ Criterion)

2. DIFFRACTION THEORY WITH ABSORBER

In the geometrical optics theory of diffraction, energy is associated with diffraction rays, which are at the edges of pieces of wave fronts. [4] The theory may be extended from diffraction by metal structures or obstacles to diffraction by absorbers by including the reflection coefficient of the absorbing surface as it effects the respective piece of the wave front near its diffracting edge. This permits the diffracting surface to be characterized as a homogeneous isotropic absorbing surface whose reflection coefficient may be a function of incidence angle and polarization. However the geometry must be such that the physical edge does not contribute appreciably to the scattering. Separate estimates of this physical effect may be required when it is significant.

3. FENCES AND IRISES

The metal iris or fence was analyzed in [1], p. 29 Figure 2 shows the direct ray and principal diffraction ray no. 1.

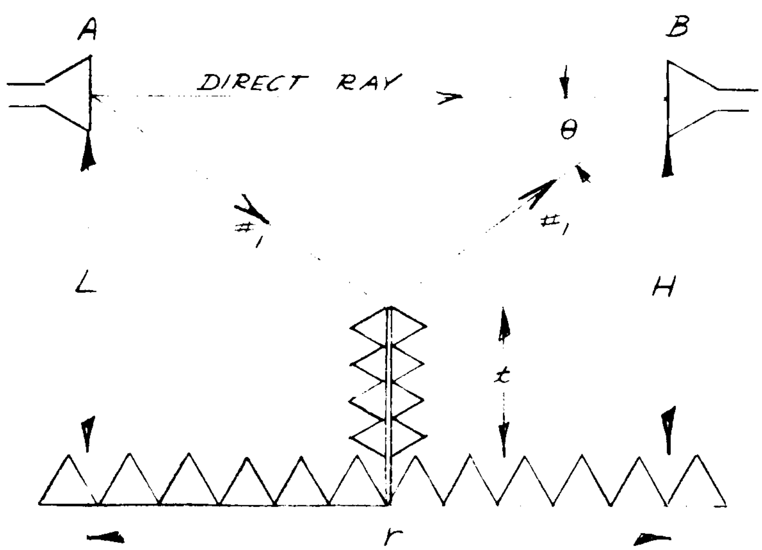

FIGURE 2

Geometry of Iris or Fence

In the metal ground plance case, image rays of source and edge also occur, but since the energy associated with them is multiplied by the small reflection coefficient of absorber these are negligible compared to the energy associated with ray no. 1 which is in turn associated with the wave front above the absorber and hence is not affected by the presence of the absorber. The ratio of total diffracted energy to direct energy is then from eq(30) of [1]

$$(2) \qquad \frac{r^2\sqrt{\lambda/2}}{8\pi\left[\left(\frac{r}{2}\right)^2+(H-t)^2\right]^{3/4}(H-t)} \simeq \frac{\sqrt{\lambda r}}{4\pi(H-t)} \quad , \quad \text{if } L = H$$

independent of the absorber characteristics.

Figures 3 and 4 show measured patterns for the case H = 10", r = 60", λ = .1233", t = 4" and t = 8.125". These have the familiar appearance of "knife-edge diffraction." The standing wave ratio of the direct and diffracted energy at L = H is read to be .6 and 2.4 db respectively. These correspond to values of 27.9db and 17db for the strength of diffracted field below the direct field assuming (as was approximately the case) isotropic antenna patterns in the plane of Figure 2. The theoretical values from eq(2) are respectively 28.8db and 17.8db. The agreement between theory and experiment indicates that the theory is quite accurate. But the concept shows a clear and significant conclusion. Namely, that use of a fence or an iris, which is a combination of two or four fences, reduces the absorbing quality of a well-designed free space room. The larger the iris, the worse the performance. Whether or not the iris is covered with absorber matters very little since the primary diffracting energy is from the edge of the terminated wave front in space. Also it is to be noted

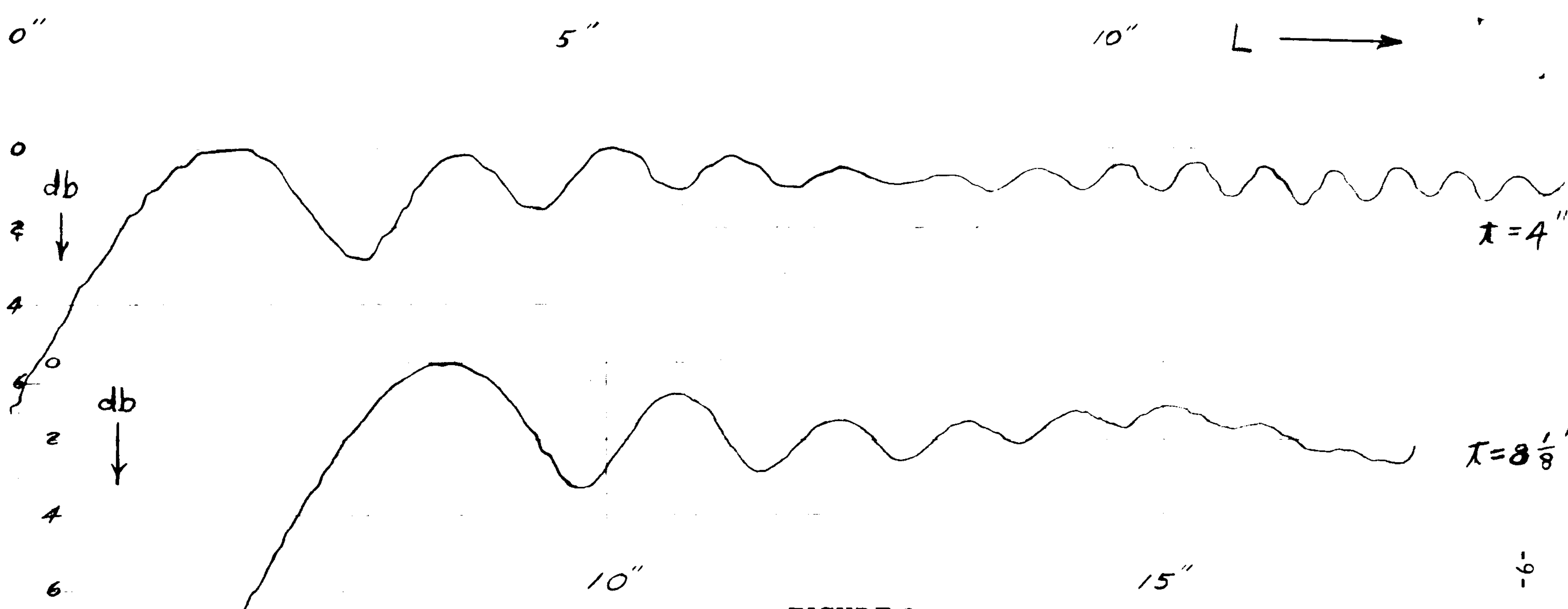

FIGURE 3

Measured Height-Gain Pattern with Fence
H = 10", r = 60", λ = .1233", t = 4"

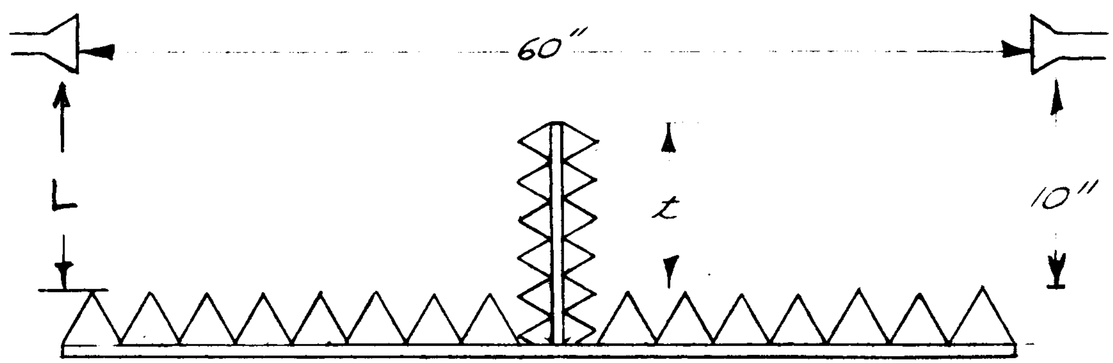

FIGURE 4

Measured Height-Gain Pattern with Fence
H = 10", r = 60", λ = .1233", t = 1.875"

that since the energy associated with the diffracted ray no. 1 of Figure 2 is large compared to the energy scattered by the actual physical top of the pieces of absorber forming the "knife-edge" the latter may be neglected. This is true only because the absorber has fairly good absorbing quality even from its edge and this fact overrides the fact that the two absorbers back-to-back forming the iris measure over 7 inches in overall lateral extent.

4. BAFFLES

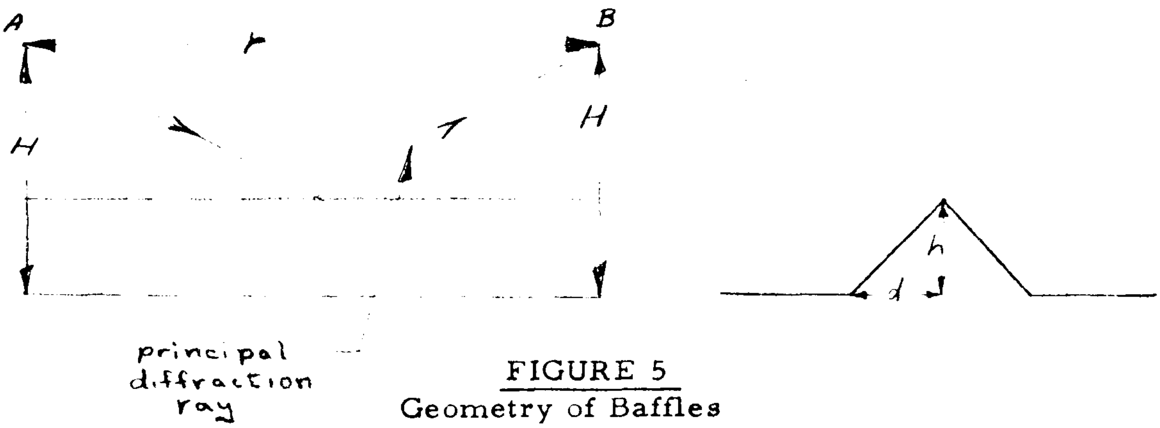

FIGURE 5
Geometry of Baffles

From eq(18) of [1] the ratio of diffracted to direct energy between istropic source and observation point, both at a height H above a ground plane, a distance r apart, and centered over a longitudinal baffle of dimensions d x h (see Figure 5 above) is given by

$$(3) \qquad \frac{Rr^2\lambda}{4\pi^2}\left(\frac{d}{h(H-h)\left[2(H-h)^2+r^2\right]^{1/4}}+\frac{h\sqrt{d^2+H^2}}{d(d^2+hH)\left[2(d^2+H^2)+r^2\right]^{1/4}}\right)^2$$

where R is the power reflection coefficient of ground plane and baffle, provided these parameters obey certain inequalities given on p. 16 of [1] which are equivalent to the baffle being larger than the first Fresnel zone. This expression was derived for the case $R = 1$ in [1], but the derivation is unchanged for a general value of R, provided that R is not too dependent on incidence angle. If R is a function of incidence angle, a value for the principal diffracting ray (Figure 5) should be used.

Now it was observed in [1] that a value of h of about H/2 and d equal to about h gives a broad optimum value for reducing diffraction. The amount of reduction then depends on λ, r, and H. For values of r and H of concern in the design of the free space room side walls of interest here, the possible improvement obtainable by the use of a baffle is about 17 db for the 8gc case and 22db for the 16gc case (p 15 of [1]). At the same time the principal diffraction ray incidence angle increases from about $70^{o} = \tan^{-1} 2.7$ to $79.5^{o} = \tan^{-1} 5.4$ in the 8gc case and from about $60^{o} = \tan^{-1} 1.7$ to $73^{o} = \tan^{-1} 3.4$ in the 16gc case. Much of this improvement is thus lost depending on the exact properties of the absorber because the absorption falls rapidly with increased incidence angle. (See Appendix B).

A further problem with the baffle arises with the dentated absorber which presents a physical edge on the baffle top (see Figure 6). In the case of the material of Figure 1, this edge is about 6" wide for an optimum baffle design. This is about 1/3 of the width of the 1st Fresnel zone in the 16gc case and 1/5 of the width of the 1st Fresnel zone in the 8gc case and hence reflecting 2/3 of the power of an infinite ground in the 16gc case and 2/5 of the power of an infinite ground in the 8gc case, but in each case at the greater incidence angle of the top of the baffle. The absorbing properties of the edge which in these

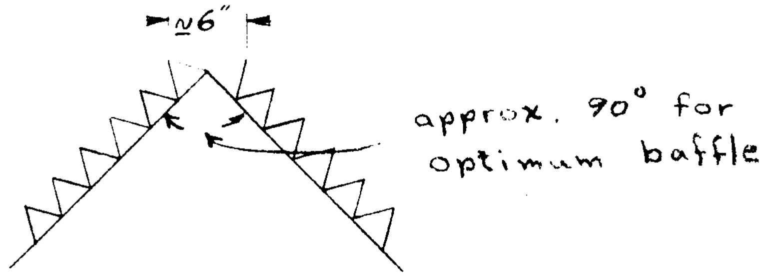

FIGURE 6

Physical Edge of Absorber Covered Baffles

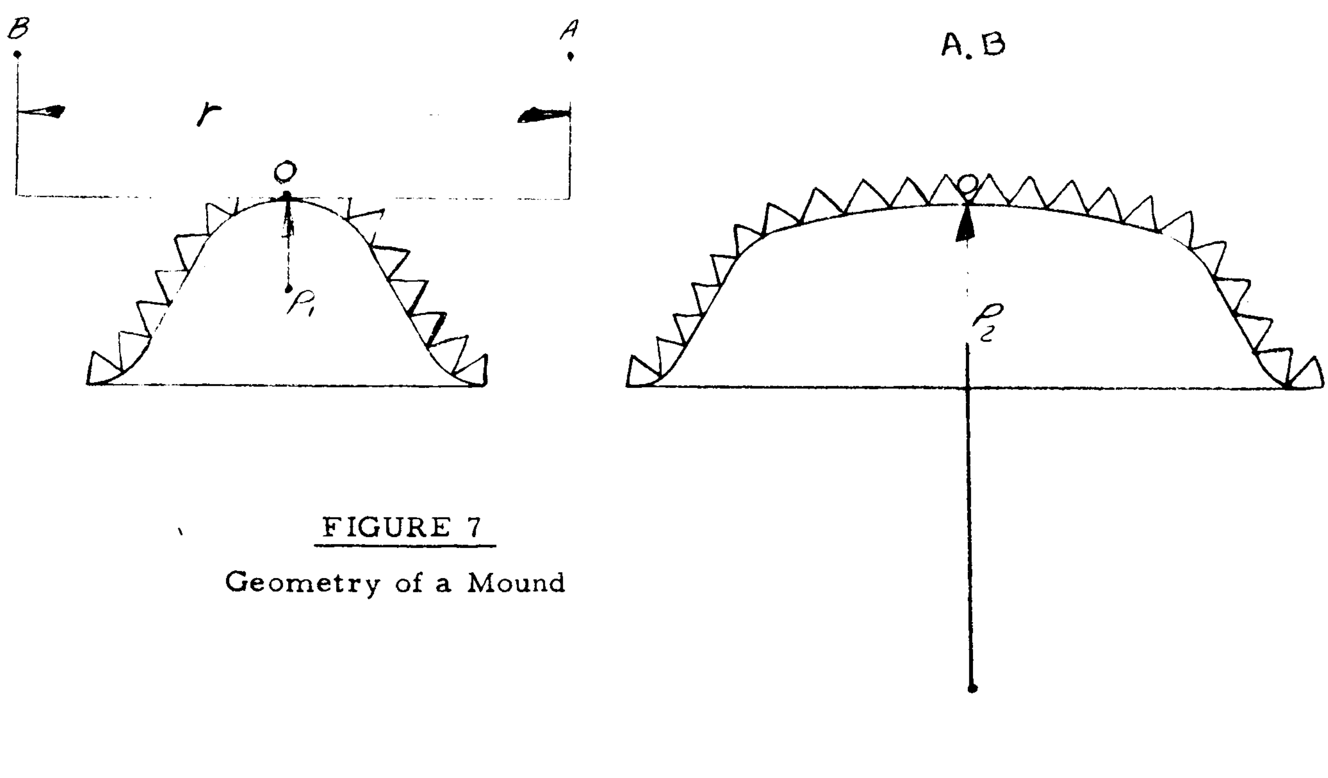

FIGURE 7

Geometry of a Mound

cases is a number of wavelengths wide will be similar to the base material without dentations and hence of the order of 20db poorer (see figures, Appendix B).

Hence while the absorber covered baffle itself may theoretically improve the absorber covered flat wall by roughly 20db, the top edge will reflect specularly as much or more than the flat wall absorber without dentations! The introduction of the baffle accordingly makes the wall reflection properties considerably poorer and should not be considered unless a method of concealing the physical edge is included. For absorber designs having larger pyramids, this edge effect is generally as bad or worse.

5. MOUNDS

The problem discussed above arises due to the abrupt change in slope at the top of the baffle. If a radius were included at the top, the exposure of the edge could be reduced or eliminated. Accordingly we have investigated the wave diffraction properties of large baffles with rounded tops. In order to derive a general formula suitable for treatment of rounded pyramidal front and back walls as well, we have assumed that the protrusion is a convex mound with maximum curvature at the top of principal radii ρ_1 and ρ_2 with curvature monotonically decreasing to the point where the mound fairs into the base wall smoothly and concavely. If the mound is larger than the first Fresnel zone, physical optics reflection formulas may be used.

The result, which is proved in the Appendix (eq(A12)) is that the power reflection is reduced because of the curvature by a factor

(4) $$P = \left(\frac{1}{1 + (r_0^{\,2}/H + H)/\rho_1}\right)\left(\frac{1}{1 + \frac{H}{\rho_2}}\right)$$

as compared to a flat ground with the same reflection coefficient. In this expression, source and observation points A and B are both a height H above the top of the mound O and separated by a distance r. The principal radii of curvature in and cross to the plane AOB are ρ_1 and ρ_2, respectively. For validity of (4), the mound must be larger than the first Fresnel zone.

It seems practical that the standard high quality dentated absorber of Figure 1 can be flexed to a radius of 1 foot but probably not much less without degrading its properties or creating the equivalent of a physical edge. For larger pyramid material the minimum value would be less.

A suggested mound design for the side walls incorporating this radius transversely and having a mound larger than the first Fresnel zone is shown in Figure 8. It includes the possibility of positioning the antennas about 1 foot off of the room center line with still improved performance.

The theoretical improvement using such a mound on each side wall is about 26db from eq(4). Such a mound is required on each of four side walls. A similar improvement is required for the back wall where d = 6' (see Table 2). In this case the pertinent equation is (A15) of the Appendix which shows that for a 1 foot radius a 20db improvement can be expected.

An improvement of more than 20db should not be sought by these means because the energy associated with the diffraction rays to the edges of the room are of that order below the specular reflection values and these room edge

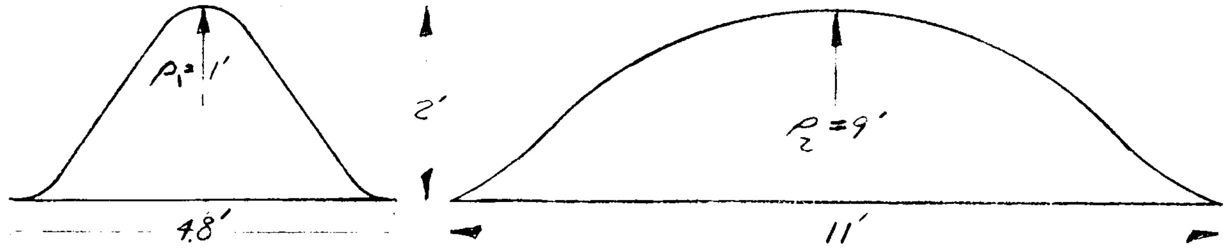

FIGURE 8a
Recommended Mound Design - Side Walls

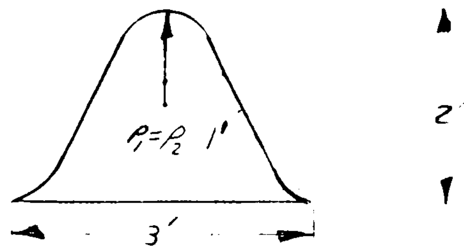

FIGURE 8b
Recommended Mound Design - Back Wall

effects can not be eliminated by any of the means discussed. Moreover as discussed on p. 6 unless the gain of the test antenna is above 20db this improvement will not actually be realized and then only when the test antenna is beamed on or near what would have been a wall specular reflection beam.

6. TILTED BACK WALL

The largest source of reflected energy in practice generally comes from the back wall because the transmitting antenna is beamed toward the specular reflection point there unless the wall is tilted or otherwise modified. The factor G_{Tr}/G_{Td} is approximately unity in (1) for this case, whereas by proper antenna design it can be made at least -20db for all other walls. We have already seen that a 20db improvement can be obtained by a bump. Can tilting the back wall do as well? This would be true if the energy within the -20db contour of the main beam of the transmitter could be reflected entirely to one side of the test antenna by a tilted wall. The limiting case occurs if the

-20db ray were reflected to the center of the test antenna. Consider the 8gc case first. Here the test antenna may be as large as 23" so the transmitting antenna 3db contour could be no closer than $\pm$ 23/2" from the center of the room. By normal beam scaling this puts the -20db contour at about 3' or more from the room center line; no reasonable wall tilt can reflect this ray behind the test antenna if the latter is only 6' from the end of the room (see Figure 9). The test antenna would have to be at least 12' from the end of the room to achieve this, and this probably wastes too large amount of the room space.

Accordingly a tilted back wall is not recommended. A similar result applies to 16gc.

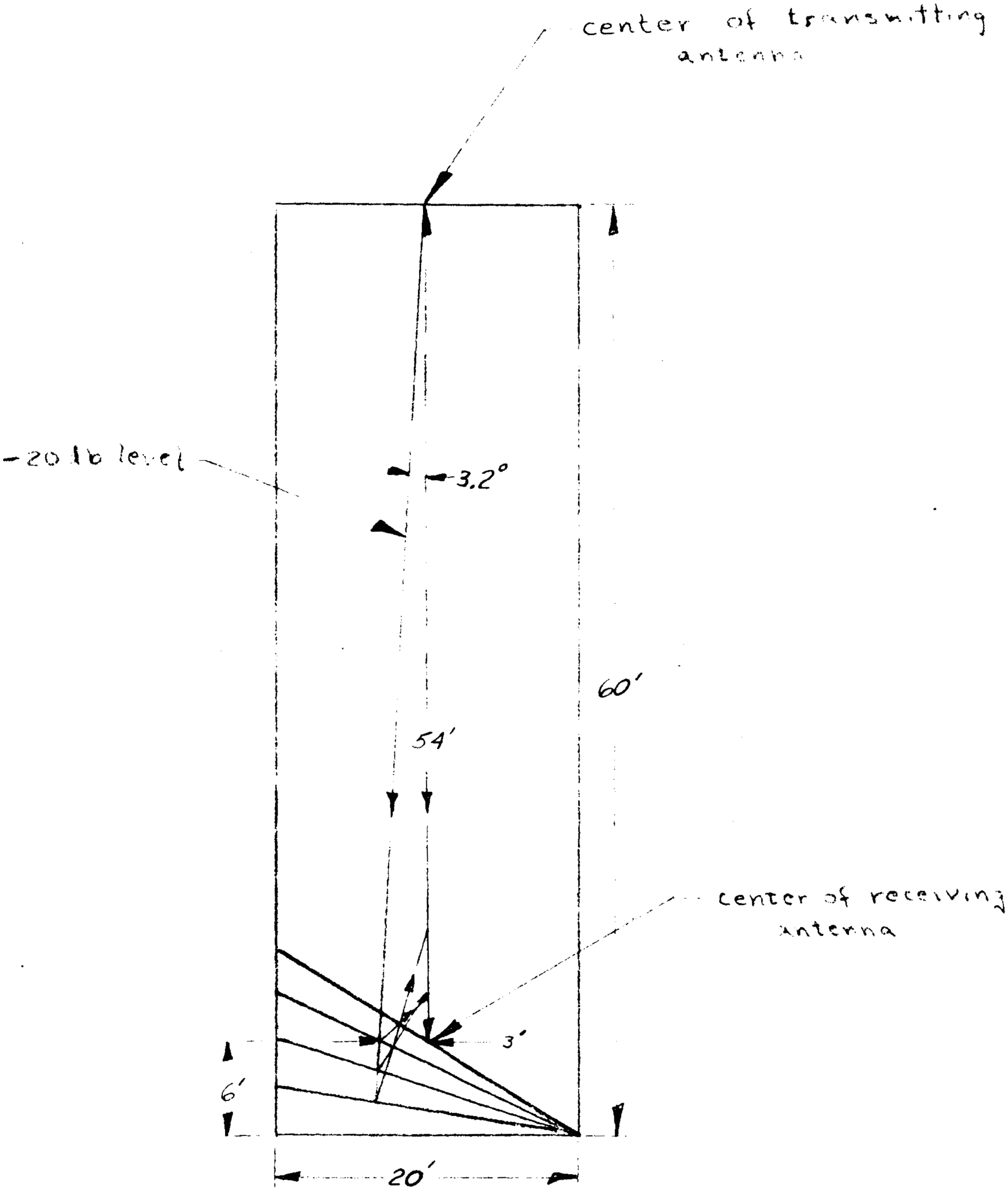

FIGURE 9

Geometry of Tilted Back Wall Showing Four Possible Positions

7. MEASUREMENTS

Measurements were made at 96gc on several different types of absorber on a number of setups. Initially baffles and fences were measured on the same 8' x 8' ground plane as was used in [1]. In every case all of the metal surfaces were completely covered with the absorber under test (see Figure 10). In the time available only the dentated absorber could be obtained in sufficient quantity to cover the ground plane, baffles, and fences. Materials of this type were obtained both from Emerson and Cuming of Canton, Mass. and the B. F. Goodrich Co. of Shelton, Conn., in sufficient quantity. Measurements on irises as described on p. 7 were successful in confirming the theory. However, measurements on baffles showed poorer results than expected which could be accounted for in at least two ways:

a) The physical baffle edge of the absorber was so wide as to be a significant source of reflection. As shown on p. 11 this effect must have been large.

b) The regular two dimensional array of identical pyramid shapes which cause reduced specular reflection may cause also some additional scattering which is not entirely diffuse and may have grating lobe effects. This effect was apparently also present since the interference patterns were not those of the expected direct and diffraction rays.

In any case baffles seemed to be quite undesirable producing more reflection than a flat ground.

Three different thin, flexible absorbers (1/2" thick) were also obtained but only one 2' x 2' piece each. Therefore a small pattern range was setup to make measurements on these materials (see Figure 11). It seemed desirable

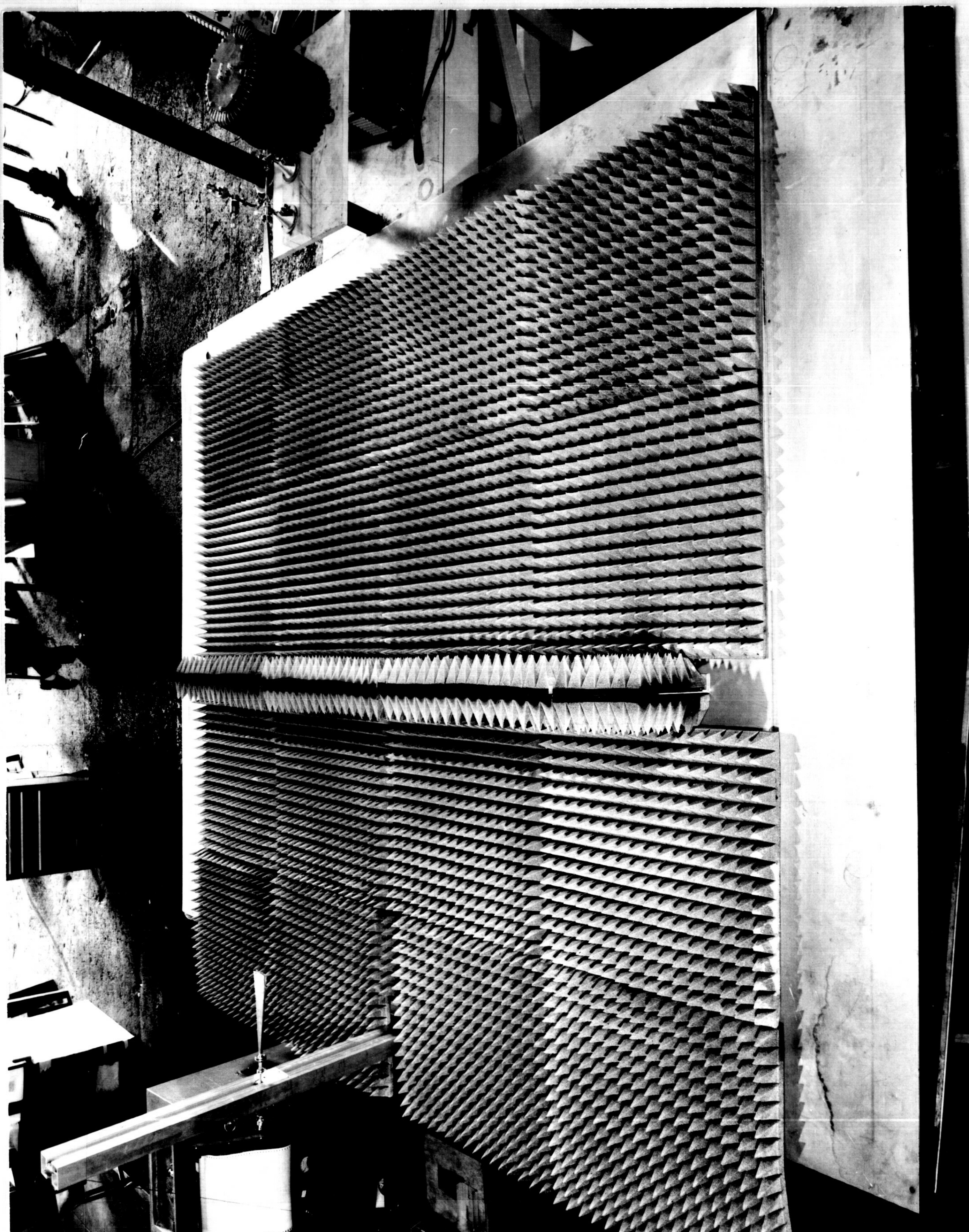

FIGURE 10 - MEASUREMENT OF ABSORBING FENCE

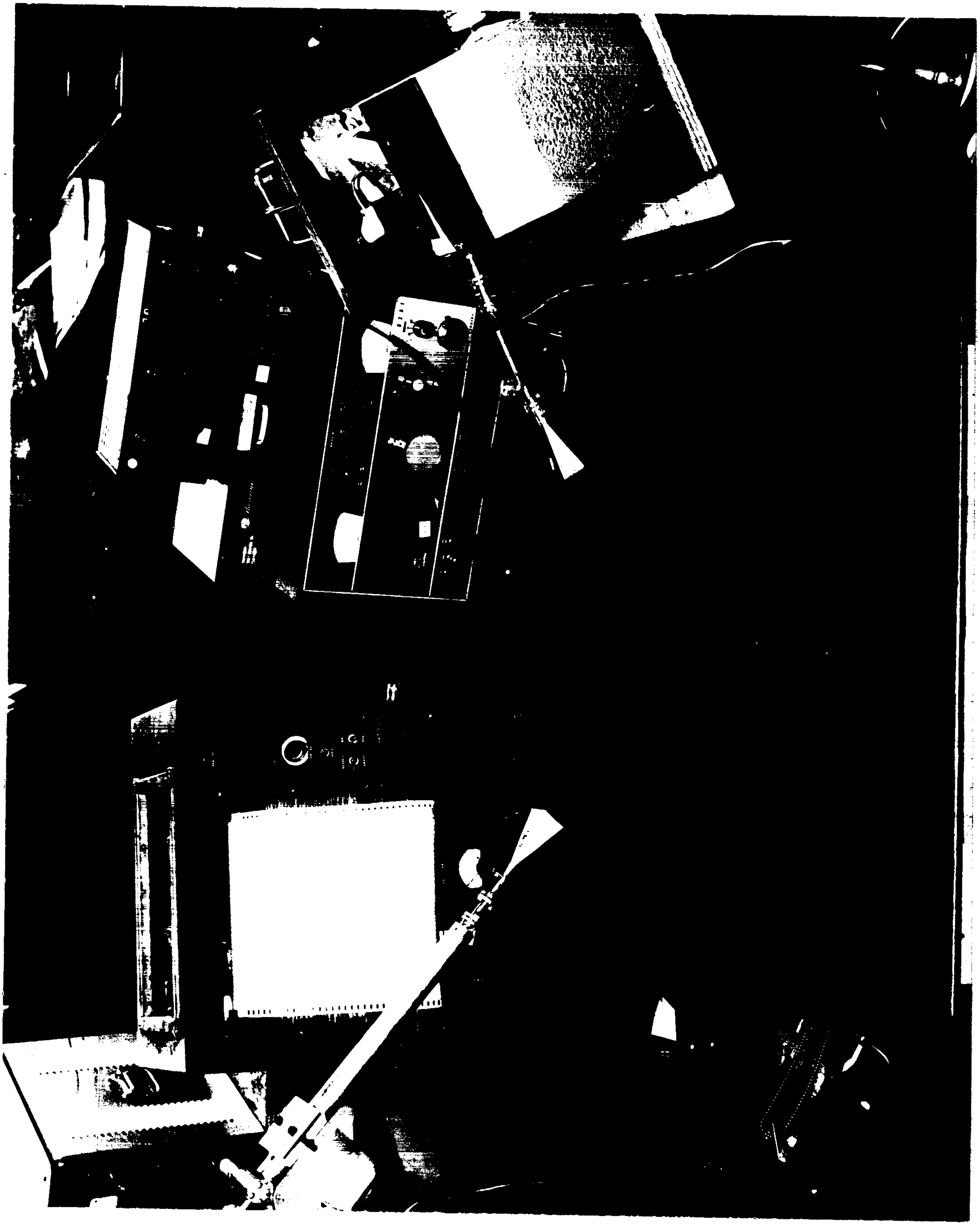

FIGURE 11 - EVALUATION OF ABSORBING MATERIAL AT 96GC

to measure the reflection coefficients versus incidence angles and polarization for each absorber. For this purpose several different methods were employed in an attempt to get the maximum dynamic range since the system was power limited. The interference method, where the reflection coefficient is estimated from the height gain curve ripple was unsatisfactory at smaller incidence angles. Accordingly also a direct substitution method was used. Since only a linear and not a circular track was used, as the height was varied from a reference height the antenna beams started pointing away from the specular reflection point. The height was varied in three ranges and the antenna pointing angles were adjusted between each range for this reason.

The results are shown on Figure B1-7 in the Appendix.

A pseudo Brewsters angle effect is visible at about 60° for the flat absorbers. Materials AN-72 and XD-AN-96A were apparently electrically identical. Material XD-AN-96B seemed a little poorer. Note that the flat backs of the dentated absorbers, which were also tested were similar in performance to the thin absorbers indicating that the materials are similar.

Attempts were made to measure the reduction in reflection by bumps by the height-gain curve or interference method. The power level was monitored and all readings could be referred to a single reference "no bump" case. Table 4 lists the results of four cases. Difficulties were experienced due to (a) inability to maintain bump curvature precisely over its length, (b) variation in absorption over different parts of the absorber, and (c) limitations in dynamic range and receiver noise effects. In Table 4 columns (7) and (9) agree reasonably well indicating that when the bump was introduced in each case the direct ray transmitted energy remained constant. From this value obtained from the reference, the reduction in the reflected energy was deduced (column 8). This agreed only qualitatively with the theoretical values (column 5), predicted from eq(A15) and corrected for the change in incidence angle due to the bump. No bumps with double curvature ($\rho_1 \rho_2 \neq 0$) could be tested since specially molded absorber would have been required. The results were sufficiently encouraging, however, so that the conclusions stated under the section "Conclusions and Recommendations" seem valid.

8. CONCLUSIONS AND RECOMMENDATIONS

1) Room stereometry or changing from a flat wall in some fashion to break up or reduce the area of the first few Fresnel zones is desirable and can give a theoretical improvement in reflection beyond that of the absorber covered flat wall of the following amounts:

a) side walls - limited by the gain of the test antenna in the specular reflection direction.

b) back wall - limited by the sum of the decibel gains of the test and transmitting antennas on the back wall.

TABLE 4

h = Height of Bump	$H = H_0 - h$	Principal Radii: ρ_1 In-plane	Principal Radii: ρ_2 Cross-plane	Incidence Angle θ	Theoretical Reduction in Scattering by Bump, db. Eq(A15) less change in reflection coefficient with incidence angle.	Height-gain Ripple SWR, db	Measured Peaks of Height-gain Curve, Relative to No-bump Reference, db	Measured Reduction in Scattering, Deduced from (6), db	Peaks of Height-gain Curve, Relative to No-bump Reference, Deduced from (8) Assuming Direct Ray Unchanged
(1)	(2)	(3)		(4)	(5)	(6)	(7)	(8)	(9)
0" (reference case)	2.5"	0"	0"	63.5°	--	5	0	0	0
.5	2.0	0	.375	68.2°	-5.6	5.6	-.5	-1	-.8
.5	2.0	.375	0	68.2°	-13.5	12	-7.	-15.5	-9.1
1.0	1.5	0	.625	73.3°	-1.1	6	-.5	-1.4	-1.1
1.0	1.5	.625	0	73.3°	-10.5	12	-9.	-15.5	-9.1

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In any case even if these sources of reflection were eliminated reflections from the edges of the room become significant. These are only about 20db below the flat side wall reflections.

2) A dentated absorber is desirable because it has both greater absorption and produces partially the effect of stereometry.

3) Irises and fences are definitely undesirable in any type of free space room regardless of the physical edge thickness of the absorber.

4) Baffles with slope discontinuities are generally undesirable and are not recommended.

5) A mound of dimensions of Figure 8a is recommended for further consideration for each of the four side walls and one of dimensions of Figure 8b is recommended for the back wall. The improvement due to these mounds should be verified at true scale frequencies with the particular absorber to be used in the free space room. The back wall treatment is particularly desirable. Theoretically an overall improvement of 20db in the wall reflection can be obtained with these mounds.

6) Alternatively a transmitting antenna can be specially designed so that energy radiated to the side wall is at least 40db below peak. Techniques which can be used for the design fof such an antenna are:

> a) absorbing tunnel, b) extreme amplitude taper, c) minimum or no feed blocking.

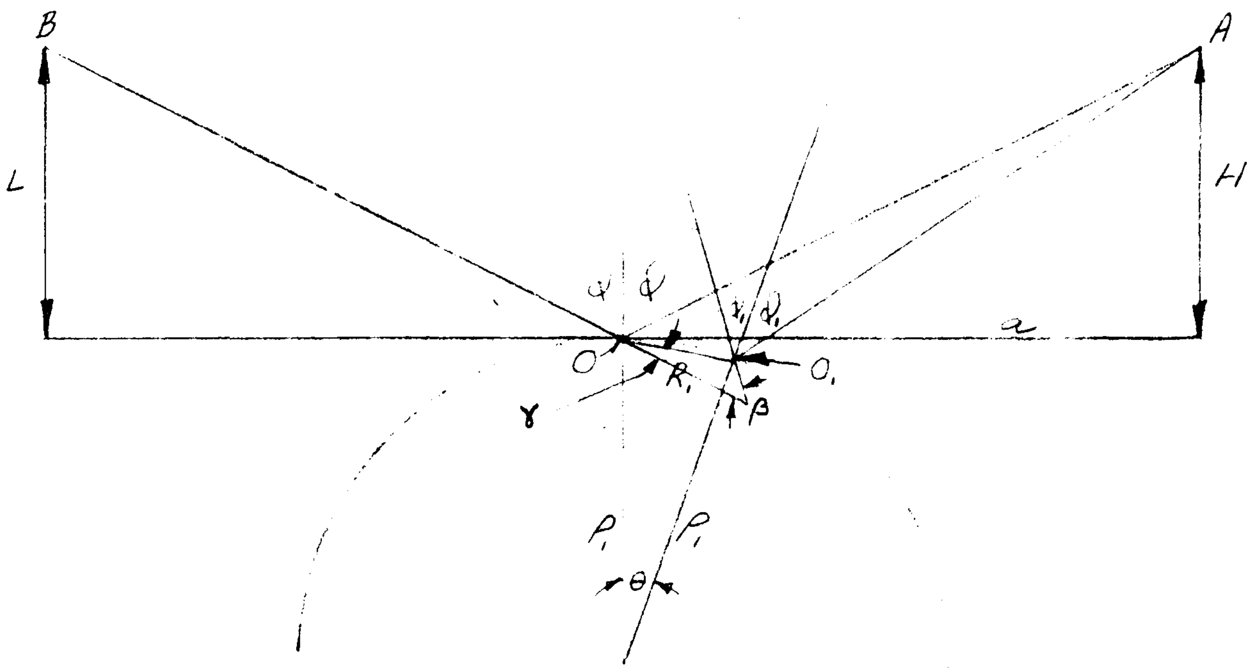
Such an antenna can be made to cover a complete waveguide band and would eliminate the need for side wall treatment leaving only one back wall mound required.

7) A mound is superior to a tilted back wall by doing the required job without taking up as much room space.

APPENDIX A - REFLECTION OF A SPHERICAL WAVE FRONT BY A SURFACE OF ARBITRARY CURVATURE

a) In-plane Image Center

FIGURE A1 - In-plane Phase Center of a Wave Front Reflected by a Circular Body



Suppose a ray is incident at angle ϕ to the stationary phase point 0 of a surface whose radius of curvature in the plane of incidence shown in Figure A1 is ρ_1. Let "a" be the distance of the source at A from the normal at 0 and let H be the distance from A to the tangent at 0. Consider a nearby ray from A whose incidence angle is ϕ_1 at the point 0_1. Let the normals at 0 and 0_1 make an angle θ at the center of curvature of the surface with respect to 0. Here θ is a small angle and all equations neglect higher order terms in θ.

The radius of curvature of the wave front after reflection is R_1 given by

(A1) $$R_1 = \frac{\sin(\gamma + \beta)}{\sin \beta}\, \rho_1 \theta$$

where

(A2) $$\gamma = \frac{\pi - \theta}{2} - \phi, \quad \beta = \theta + \phi - \phi_1, \quad \phi_1 = \tan^{-1}\left(\frac{a - \rho_1 \theta}{H}\right) - \theta \quad .$$

As $\theta \to 0$

(A3) $$\gamma \to \frac{\pi}{2} - \phi, \quad \beta \to 2\theta + \frac{\rho_1 H \theta}{a^2 + H^2}, \quad \sin(\gamma + \beta) \to \frac{H}{\sqrt{a^2 + H^2}}$$

and

(A4) $$R_1 \to \frac{\rho_1 H \sqrt{a^2 + H^2}}{2(a^2 + H^2) + \rho_1 H} \quad .$$

b) <u>Cross-plane Image Center</u>

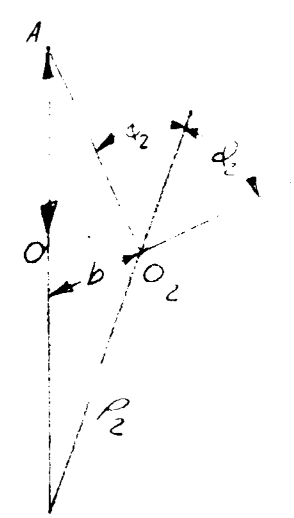


FIGURE A2
Cross-plane Phase Center of a Wave Front Reflected by a Circular Body

Consider the ray from A incident at a point O_2 near O in the transverse plane where the radius of curvature is ρ_2. The incidence angle projected in the plane AOO_2 is ϕ_2. The reflected ray projects back a distance b from O_2 to the line $\overline{AO}$ where b is given by

(A5) $$b = \frac{\rho_2}{2 + \frac{\rho_2}{H}}$$

to first order as $\phi_2 \to 0$. b is the projection in the plane AOO_2 of the reflected wave front's radius of curvature R_2 in the cross-plane. In fact

(A6) $$R_2 \cos \phi = b$$

or

(A7) $$R_2 = \frac{\rho_2 \sqrt{a^2 + H^2}}{2H + \rho_2} .$$

An alternative derivation of R_1 and R_2 on a little more sophisticated level is the following: Let the source be at (a, 0, H) in an (x, y, z) coordinate system with the point O at the origin. The series expansion of the equation of the surface is

(A8) $$z = -\frac{x^2}{2\rho_1} - \frac{y^2}{2\rho_2} + \dots .$$

The unit normal at the point (x, y, z) is

(A9) $$\bar{n} = \frac{\left(\frac{x}{\rho_1}, \frac{y}{\rho_2}, 1\right)}{\sqrt{1 + (x^2/\rho_1^{\,2}) + (y^2/\rho_2^{\,2})}} \;.$$

If $\bar{a}$ is a vector in the incidence direction

(A10) $$\bar{a} = (a-x,\ y,\ H-z)\ .$$

The reflected ray is

(A11) $$2(\bar{n}\cdot\bar{a})\bar{n} - \bar{a} \simeq \left(\left(\frac{2H}{\rho_1} + 1\right)x - a,\ \left(\frac{2H}{\rho_2} + 1\right)y,\ \frac{2ax}{\rho_1} + H \right)$$

where all terms quadratic in x and y are omitted. The phase center of the reflected rays to first order (ξ, η, δ) is then given by

(A12) $$\frac{x - \xi}{\frac{2Hx}{\rho_1} + x - a} = \frac{y - \eta}{\left(\frac{2H}{\rho_2} + 1\right)y} = \frac{z - \delta}{H + \frac{2ax}{\rho_1}}$$

where (ξ, η, δ) make (A12) correct to first order for small x and y. The values of R_1 and R_2 are equal respectively then to $\sqrt{\xi^2 + \delta^2}$ and $\sqrt{\eta^2 + \delta^2}$. The result is the same as equations (A4) and (A7).

Now from eq(26) of [5], the field at the observation point B at a distance p from the stationary phase point will be

(A13) $$u_p = u(0)e^{-jkp}\sqrt{\frac{R_1R_2}{(R_1+p)(R_2+p)}}$$

where u(0) is the field at O. The ratio P of the power radiated at B relative to what would have been radiated if the reflector were flat (and R_1 and R_2 equal to $\sqrt{a^2+H^2}$) is given by

(A14) $$P = \frac{R_1R_2}{(R_1+p)(R_2+p)}\frac{(\sqrt{a^2+H^2}+p)^2}{(a^2+H^2)} .$$

Substituting (A1) and (A7) into (A14) yields

(A15) $$P = \frac{\left(\rho_1\rho_2 H\sqrt{a^2+H^2}+p)^2\right)\Big/\left((2H+\rho_2)\left[2(a^2+H^2)+\rho_1 H\right]\right)}{\left(\rho_1 H\sqrt{a^2+H^2})/(2(a^2+H^2)+\rho_1 H)+p\right)\left(\rho_2\sqrt{a^2+H^2})/(2H+\rho_2)+p\right)} .$$

If the observation point and source are symmetrically located, as in the case of the side walls then

(A16) $$p = \sqrt{a^2+H^2}$$

and (A15) becomes

(A17) $$P = \frac{\rho_1\rho_2 H}{(\rho_1 H + a^2 + H^2)(\rho_2 + H)} .$$

If the observation point and source are both located on the normal to the wall as on the front and back walls then $a = 0$

(A18)
$$P = \frac{\rho_1 \rho_2 (p+H)^2}{\left[(\rho_1 + 2p)H + p\rho_1\right]\left[(\rho_2 + 2p)H + p\rho_2\right]} \; .$$

In this case normally also the bump would be symmetric with

(A19)
$$\rho_1 = \rho_2 \; .$$

Then

(A20)
$$P = \left[\frac{\rho_1 (p+H)}{(\rho_1 + 2p)H + p\rho_1}\right]^2 , \quad \rho_1 = \rho_2 \; .$$

If (A20) is written in the form

(A21)
$$P = \left(\frac{1}{1 + (2/\rho_1(\frac{1}{p} + \frac{1}{H})}\right)^2 , \quad \rho_1 = \rho_2 \; .$$

It is clear that P is minimized by minimizing $\rho_1 = \rho_2$ and maximizing p and H subject to whatever other constraints are imposed on these variables.

APPENDIX B - MEASUREMENTS OF COMMERCIALLY AVAILABLE ABSORBER AT 96GC

Five different commercially available absorbers were tested at 96gc on a small pattern range shown in Figure 11. The basic test method was by substitution of the absorber for a metal ground and recording the power drop. Direct energy from transmitting to receiving horns was reduced by using gain standard horns of about 25db gain each. The incidence angle was varied by changing the height of the receiving antenna above the ground plane. Since this also shifted the peaks of both transmitting and receiving beams away from the specular reflection point an error was introduced on either side of a reference height where transmitting and receiving antennas both had the same height. To minimize this effect these "height-gain" curves were repeated in three setups, the antennas being repointed after each case at a new reference height. The data, plotted in Figures B1-7 shows three intervals of data with some obvious inconsistencies at the limits. Since the setup was power limited, in the case of dentated or pyramidal absorber which scatters as well as absorbs no reliable readings were obtained but an estimate of the upper bound on absorption was determined. from the receiver noise level.

Materials tested were:

Figure No.	Manufacturer	Absorber Type
B1	E and C	AN-72
B2	E and C	XD-AN-96A
B3	E and C	XD-AN-96B
B4	E and C	CV-4 face down
B5	E and C	CV-4 normal position
B6	BFG	VHP-2 face down
B7	BFG	VHP-2 normal position

E and C = Emerson and Cuming, Inc., Canton, Mass.

BFG = B. F. Goodrich Corp., Sponge Rubber Products Div., Shelton, Conn.

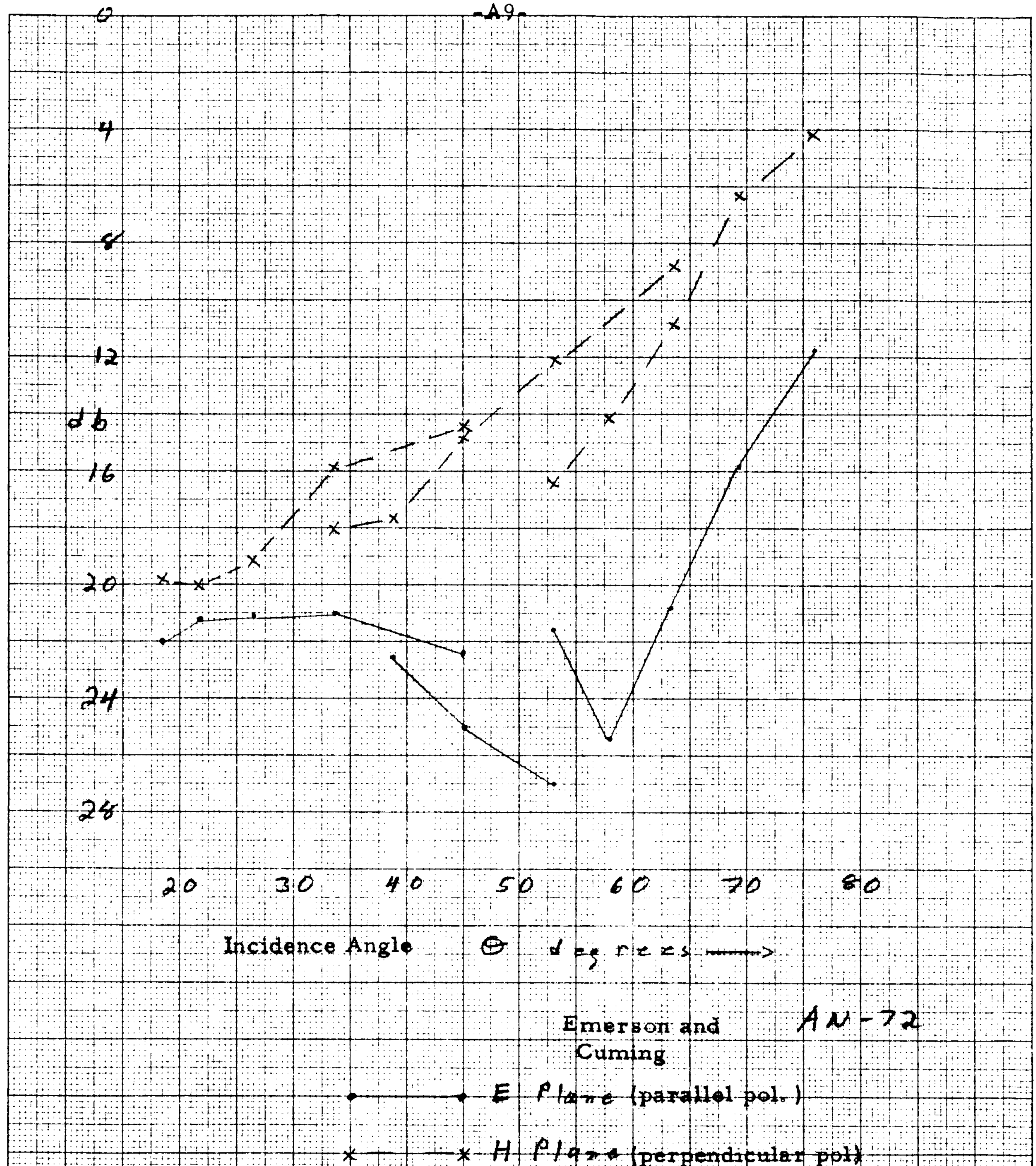


FIGURE B-1
Reflection Coefficient at 96 gc

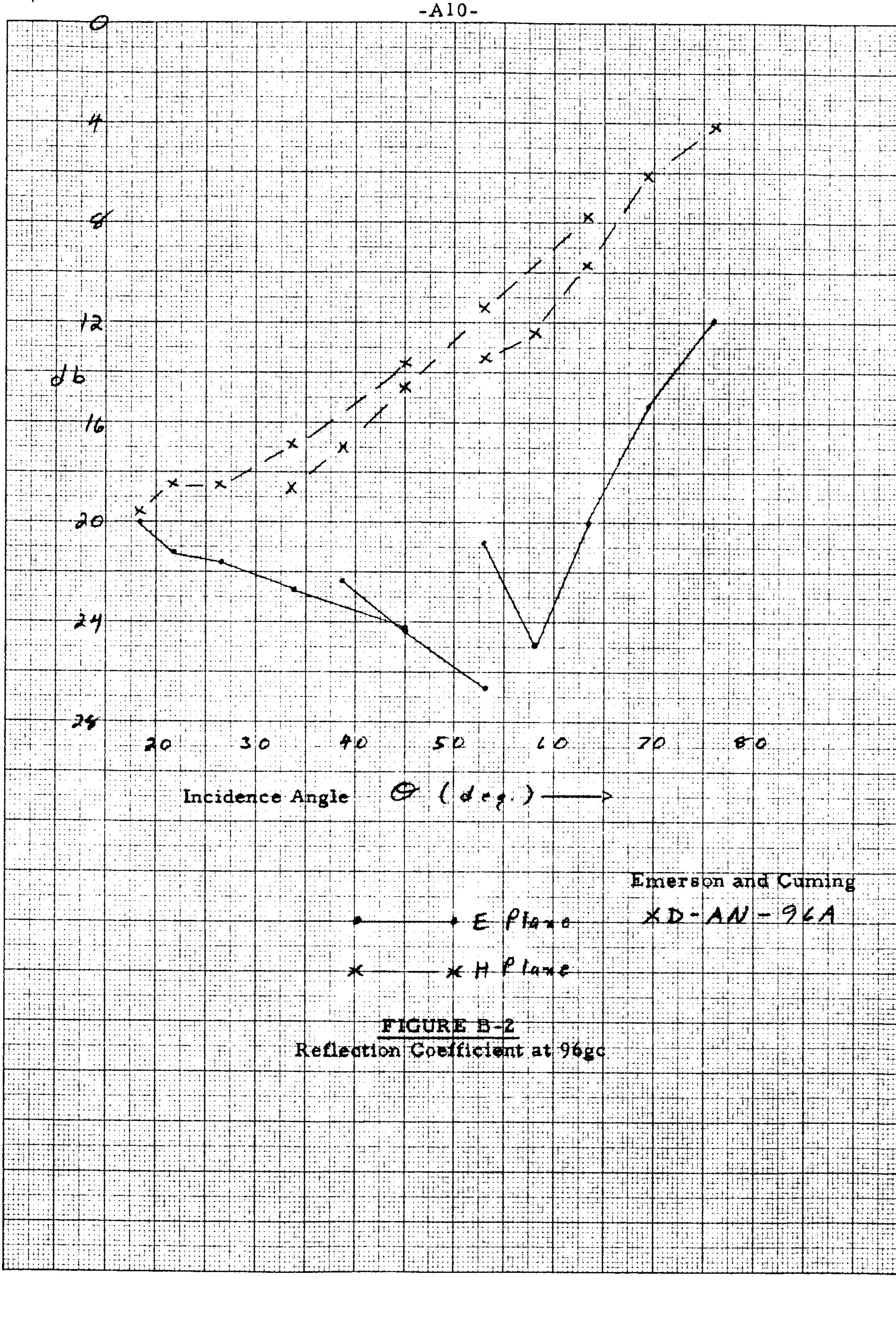


FIGURE B-2
Reflection Coefficient at 96gc

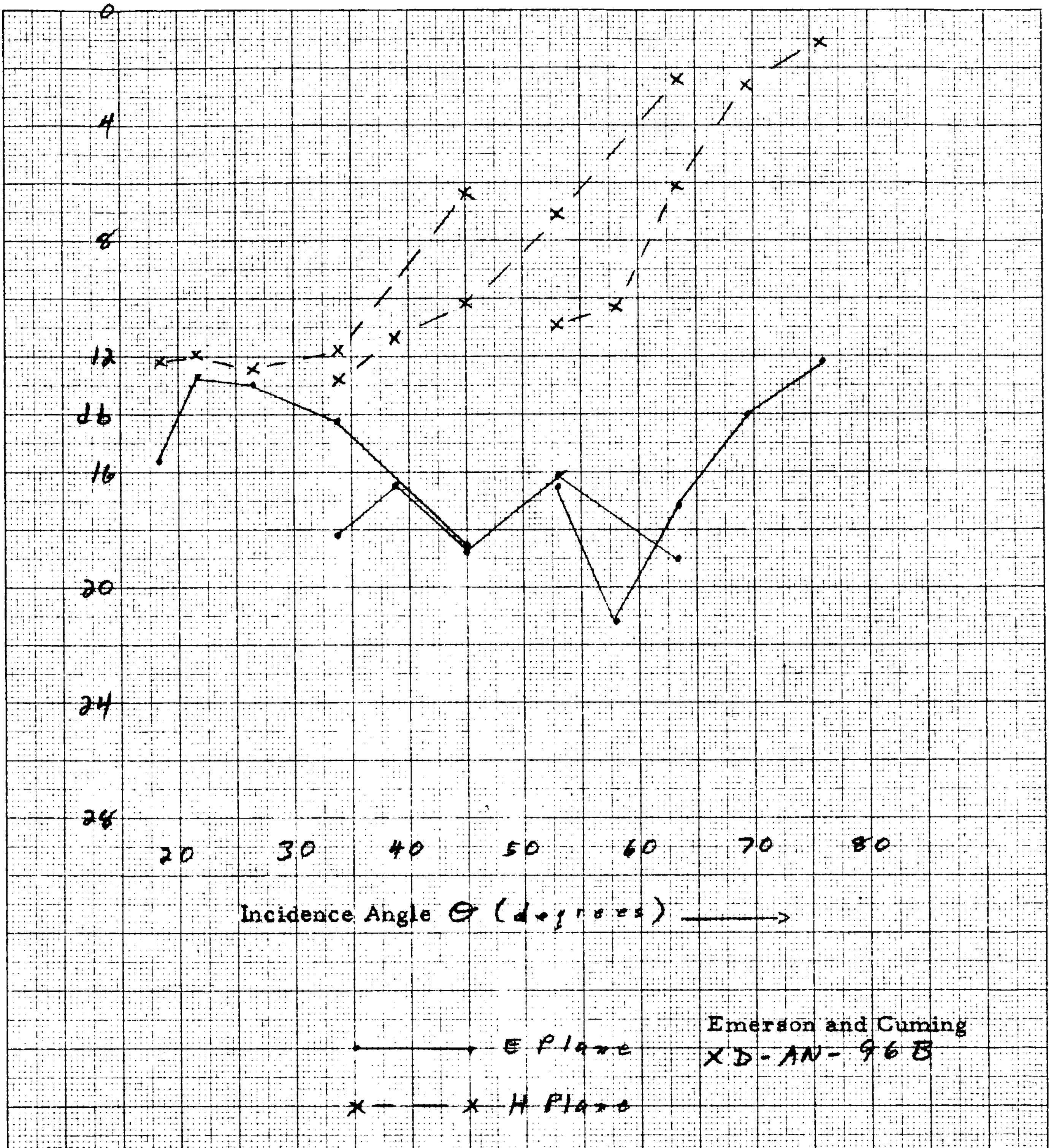


FIGURE B-3
Reflection Coefficient at 96gc

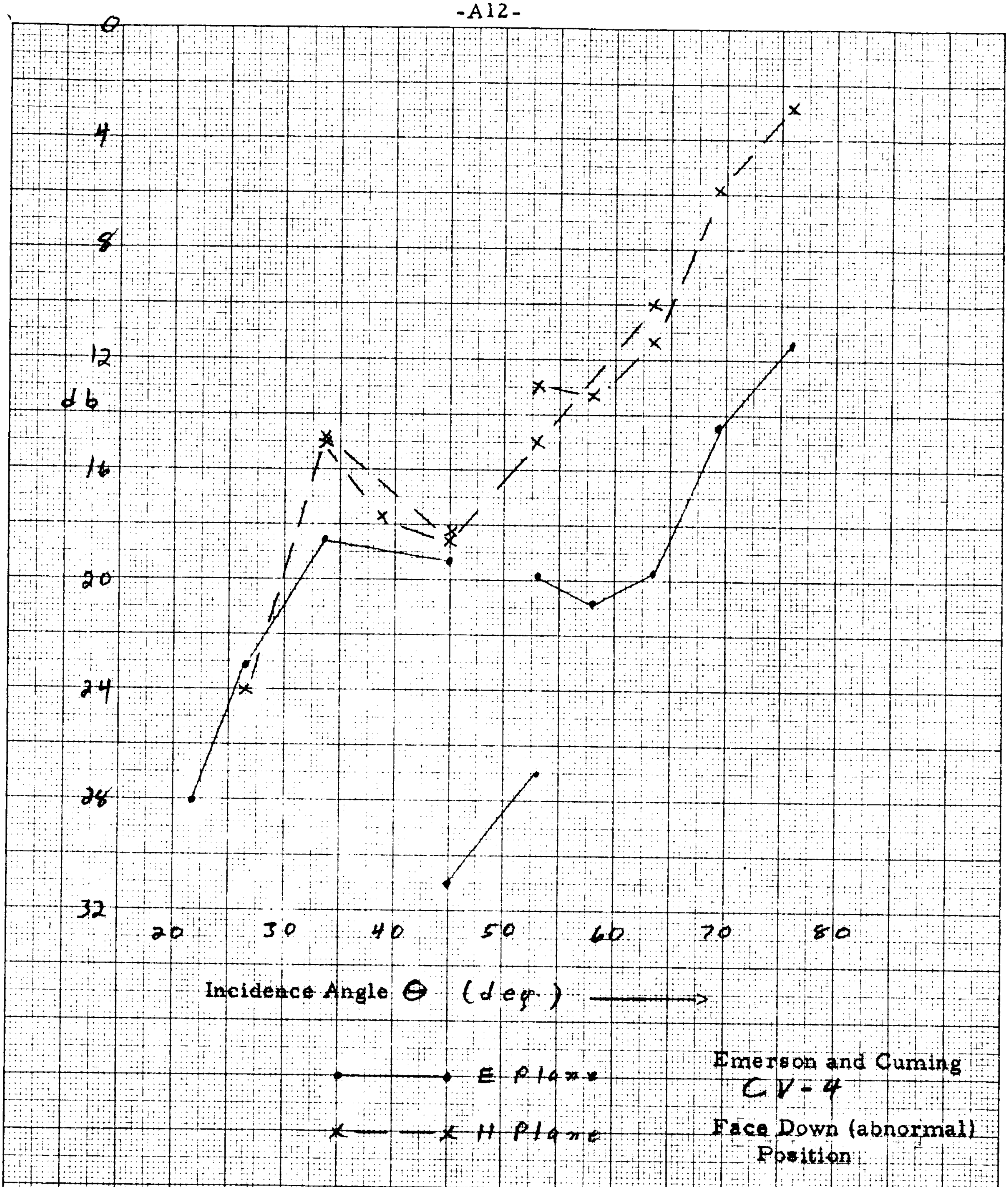


FIGURE B-4

Reflection Coefficient at 96gc

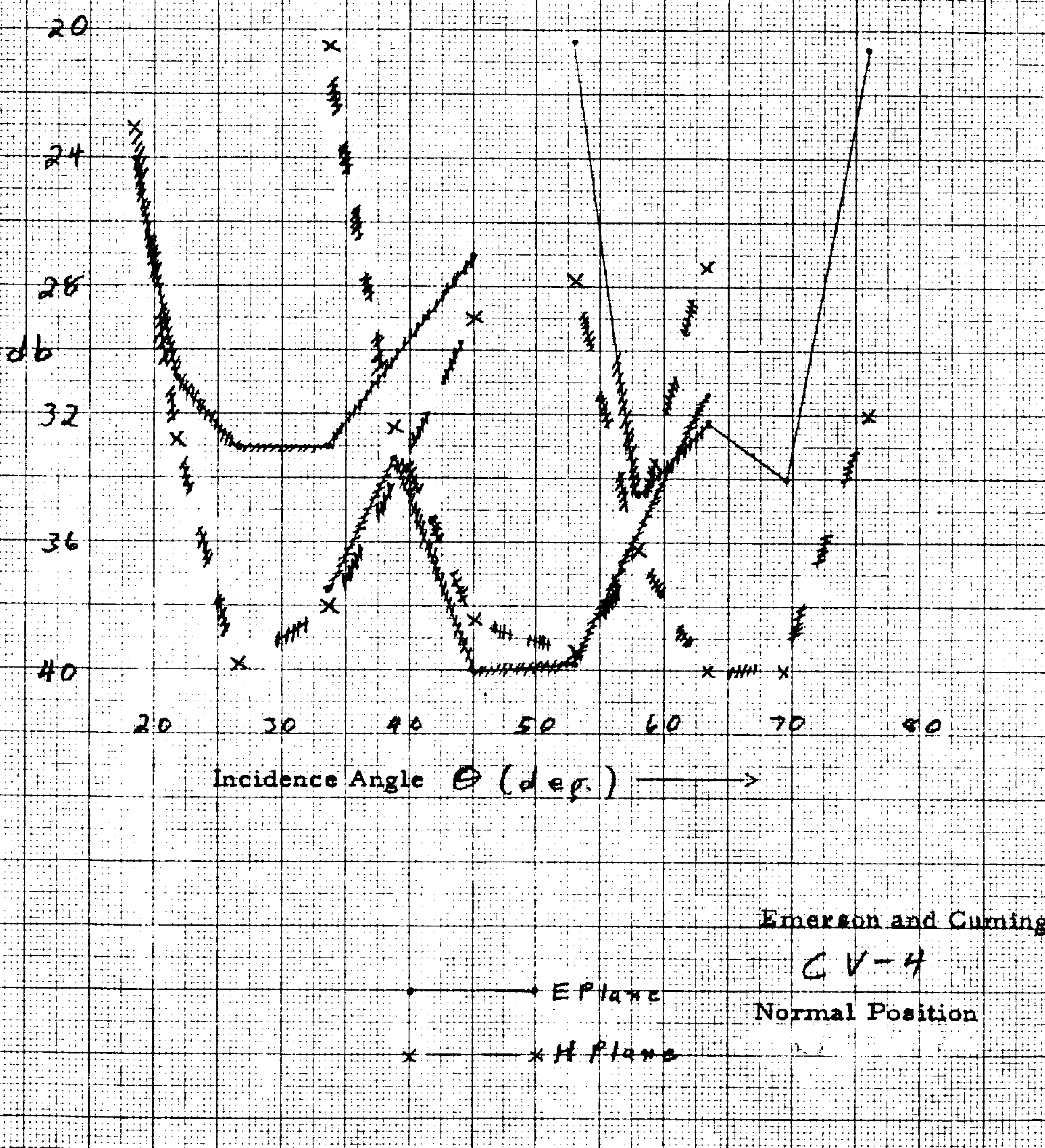


FIGURE B-5

Reflection Coefficient at 96gc

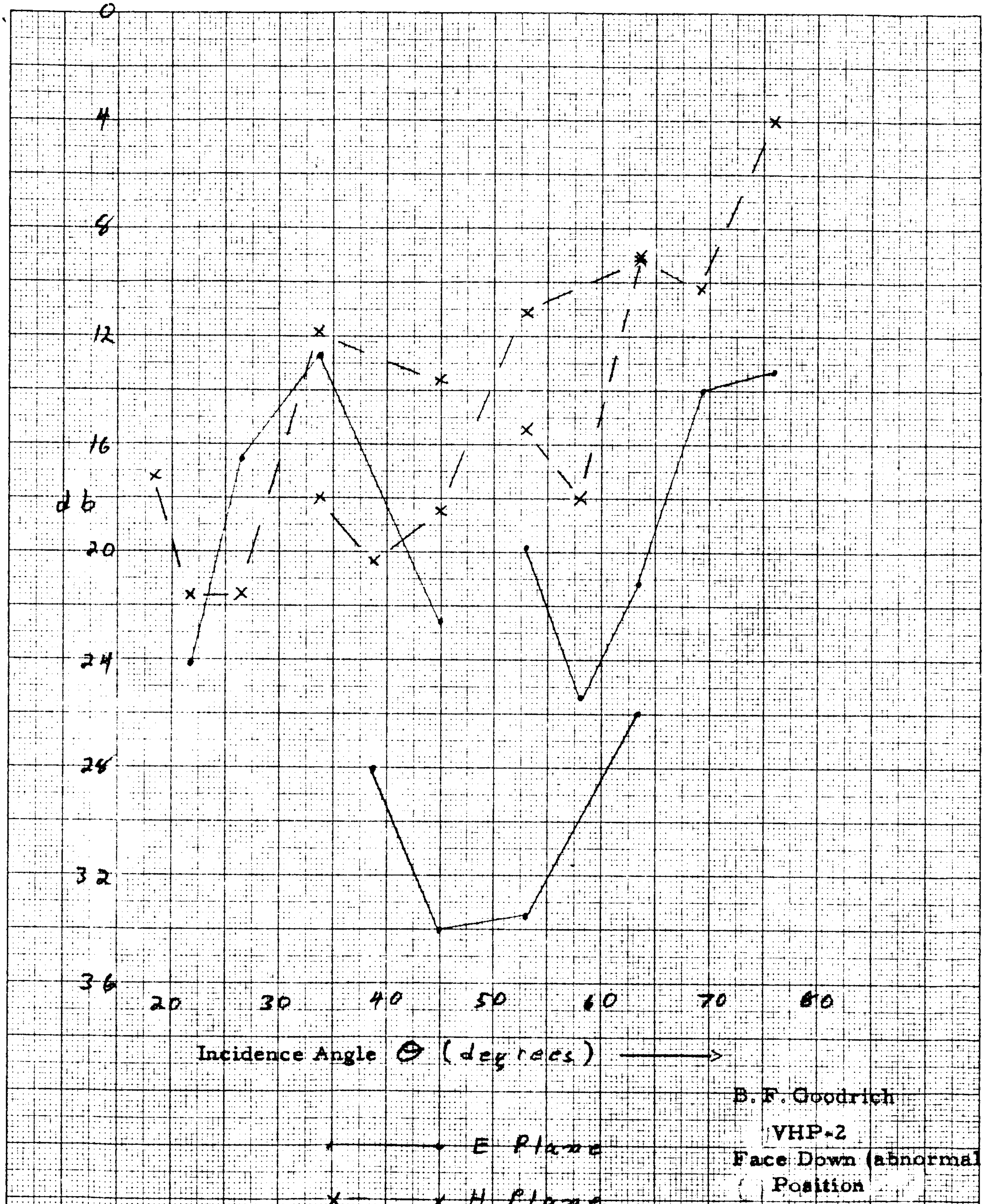


FIGURE B-6
Reflection Coefficient at 96gc

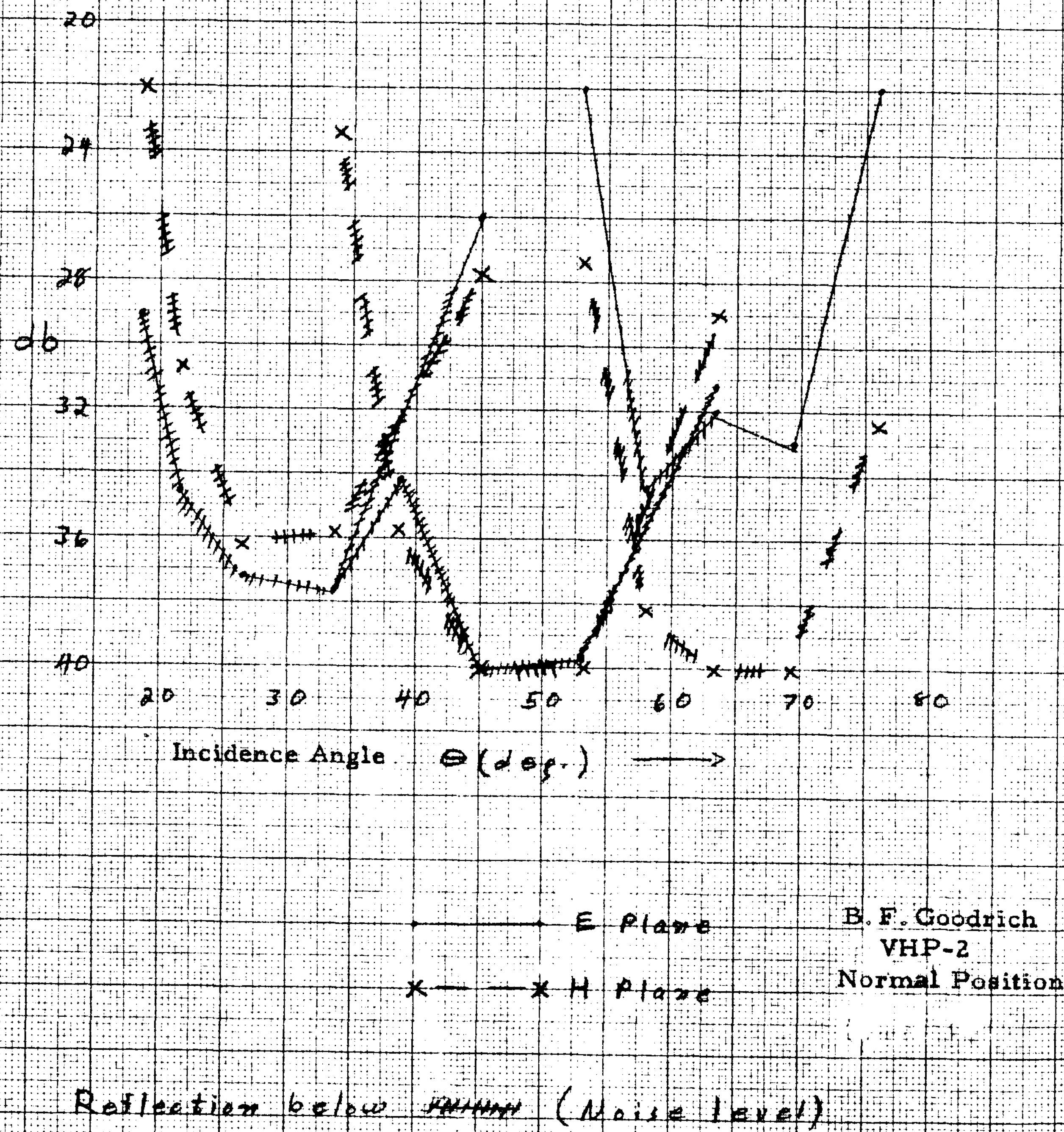


FIGURE B-7

Reflection Coefficient at 96gc

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